

Calculators and mobile phones are not allowed.

Answer all of the following questions.

1. Evaluate the following limits, if they exist:

(a) $\lim_{x \rightarrow \infty} (1 + 2x)^{e^{-x}}$

(b) $\lim_{x \rightarrow 1^+} \left(\frac{1}{x-1} - \frac{x}{\ln x} \right)$

(5 points each)

2. Evaluate the following integrals:

(a) $\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx$

(b) $\int \frac{\sin^5 x}{\sqrt{\cos x}} dx$

(c) $\int \frac{6x^2 - 3x + 1}{(4x+1)(x^2+1)} dx$

(d) $\int \frac{dx}{4 + \sin x}$

(5 points each)

3. Determine whether the following integral converges, and if it converges, find its value.

$$\int_2^{\infty} \frac{dx}{x\sqrt{x^2+4}}$$

(5 points)

4. Let the curve C be given parametrically by

$$x = t + \sin t, \quad y = 1 - \cos t \quad \left(0 \leq t \leq \frac{\pi}{2} \right)$$

(a) Find the slope of tangent line to C at $t = \frac{\pi}{3}$.

(2 points)

(b) Calculate the length of the curve C .

(3 points)